

A Phenomenological Unified Framework: Frequency-Dependent Yukawa Potentials Linking Gravity and Residual Nuclear Interactions via an Empirical Constant Relation

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(Dated: April 11, 2026)

We present a phenomenological scalar field model that attempts to connect the long-range gravitational interaction and the short-range residual strong nuclear force through a generalized Yukawa-type equation with interaction-specific characteristic frequencies Ω . For gravity, $\Omega = H_0$ yields effectively infinite range; for the residual nuclear force, $\Omega \approx m_p c^2/\hbar$ yields \sim fm range. The model is motivated by a striking empirical relation (accurate to 0.037% using Planck 2018 H_0 , and 0.584% with 2025 combined CMB data) linking G , H_0 , and atomic/quantum constants. We derive the force law, confront severe experimental constraints on massive gravity/Yukawa modifications, and discuss limitations in reconciling with general relativity and QCD.

I. INTRODUCTION

The unification of gravity with other fundamental interactions remains an open challenge. Here we explore a purely phenomenological scalar toy model inspired by Yukawa potentials, where interaction range is set by a characteristic frequency Ω . This heuristic highlights the $\sim 10^{42}$ hierarchy in force ranges via vastly different Ω scales.

The model is anchored by a recently quantified empirical relation between fundamental constants [1].

II. EMPIRICAL CONSTANT RELATION

We consider the relation

$$\frac{G}{H_0} = \frac{c^2}{\sqrt{2}\hbar} \frac{4}{3}\pi \left(\frac{\hbar^2 \varepsilon_0}{m_p e^2} \right)^3 \quad (1)$$

or equivalently,

$$G = \frac{c^2 H_0}{\sqrt{2}\hbar} \cdot \frac{4}{3}\pi \left(\frac{\hbar^2 \varepsilon_0}{m_p e^2} \right)^3. \quad (2)$$

Using CODATA 2022 values [2] and Planck 2018 CMB-inferred $H_0 = 67.66 \pm 0.42$ km/s/Mpc ($\approx 2.193 \times 10^{-18}$ s⁻¹), the right-hand side reproduces measured $G = 6.67430 \times 10^{-11}$ m³ kg⁻¹ s⁻² to within 0.037% relative discrepancy. With the latest 2025 combined CMB data (Planck + SPT + ACT) $H_0 = 67.24$ km/s/Mpc, the discrepancy is -0.584%. Using local SH0ES $H_0 \approx 73$ km/s/Mpc, it rises to $\sim 8\%$, suggesting the relation favors the lower (CMB) value and may probe physics underlying the Hubble tension.

The term $\left(\frac{\hbar^2 \varepsilon_0}{m_p e^2} \right)$ has dimensions of length, resembling a modified reduced Compton radius for the proton in an electromagnetic context; the factor $\frac{4}{3}\pi$ suggests a spherical volume interpretation.

III. THE PHENOMENOLOGICAL FIELD EQUATION

We propose a linear scalar field equation

$$\nabla^2 \varphi - \kappa^2 \varphi = 4\pi \frac{G\Omega}{H_0} \rho, \quad (3)$$

with $\kappa = \Omega/c$ and range c/Ω . For a point source of mass M at the origin, the static solution is

$$\varphi(r) = \frac{G\Omega M}{H_0 r} e^{-r\Omega/c}. \quad (4)$$

The resulting force on a test mass m is

$$F = -m\nabla\varphi = -\frac{G\Omega M m}{H_0 r^2} \left(1 + \frac{r\Omega}{c} \right) e^{-r\Omega/c} \hat{r}. \quad (5)$$

In the limit $\Omega \rightarrow 0$ (or $r\Omega/c \ll 1$), this reduces to Newton's law $F = -GMm/r^2$.

IV. APPLICATION TO SPECIFIC INTERACTIONS

A. Gravity

Set $\Omega^{\text{grav}} = H_0 \approx 2.2 \times 10^{-18}$ Hz. Then $\Omega/H_0 = 1$, the prefactor simplifies to G , and $\kappa \approx H_0/c$ yields range $\sim c/H_0 \approx 4.5$ Gpc (Hubble radius). The potential approximates Newtonian gravity on sub-cosmological scales.

However, this implies an effective graviton mass $m_g \approx \hbar H_0/c^2 \approx 10^{-33}$ eV, corresponding to Yukawa suppression at Gpc scales. Gravitational-wave propagation (LIGO/Virgo/KAGRA) constrains $m_g \lesssim 10^{-23}$ eV [3], binary pulsars yield $m_g \lesssim 10^{-20}$ eV [4], and large-scale structure/cosmic convergence analyses set $m_g \lesssim 10^{-23}$ eV or tighter.

B. Residual Strong Nuclear Force

For the residual interaction between nucleons (mediated effectively by pions), set $\Omega^{\text{strong}} = m_p c^2 / \hbar \approx 1.4 \times 10^{24}$ Hz, yielding range $c/\Omega \sim 1.4$ fm, consistent with nuclear force phenomenology. The enormous prefactor $G\Omega/H_0 \sim 10^{42}G$ would need separate tuning to match observed binding strengths; this application is purely illustrative and does not describe fundamental QCD (gluons, confinement, asymptotic freedom).

V. DISCUSSION AND OUTLOOK

The frequency-dependent range provides an intuitive picture of force hierarchies but remains a toy model. Key

challenges include:

1. Lack of tensor structure needed for GR (gravitational waves, light bending).
2. No derivation from fundamental theory; the empirical relation, while numerically compelling, is ad-hoc.
3. No quantitative match for strong-force coupling strength.

Future work could explore derivations in emergent gravity or quantum-gravity contexts, test the empirical relation's robustness with updated H_0 , or seek screening mechanisms to evade bounds.

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